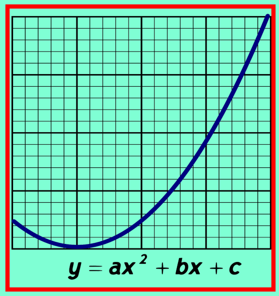
Math 125

Fall 2021

Lecture 27



Given

$$\begin{cases} 2x - 3y = 13 \\ 3x + 2y = 0 \end{cases}$$

Solve by Cramer's rule.

$$D = \begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = 2(2) - 3(-3) = \boxed{13}$$

$$D_x = \begin{vmatrix} 13 & -3 \\ 0 & 2 \end{vmatrix} = 13(2) - 0(-3) = \boxed{26}$$

$$D_y = \begin{vmatrix} 2 & 13 \\ 3 & 0 \end{vmatrix} = 2(0) - 3(13) = \boxed{-39}$$

$$x = \frac{D_x}{D} = \frac{26}{13} = \boxed{2}$$

$$y = \frac{D_y}{D} = \frac{-39}{13} = \boxed{-3}$$

$$\{ (2,-3) \}$$

Final Ans
$(x, y) = (2, -3)$

Now evaluating the determinant of a 3x3 Matrix.

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1 \begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} \overset{\text{Always}}{-} b_1 \begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1 \begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$$

Evaluate

$$\begin{vmatrix} 1 & 2 & 3 \\ -2 & 4 & 5 \\ 0 & 1 & 6 \end{vmatrix} = 1 \begin{vmatrix} 4 & 5 \\ 1 & 6 \end{vmatrix} - 2 \begin{vmatrix} -2 & 5 \\ 0 & 6 \end{vmatrix} + 3 \begin{vmatrix} -2 & 4 \\ 0 & 1 \end{vmatrix}$$

$$= 1(4\cdot 6 - 1\cdot 5) - 2(-2\cdot 6 - 0\cdot 5) + 3(-2\cdot 1 - 0\cdot 4)$$

$$= 1(24-5) - 2(-12-0) + 3(-2-0)$$

$$= 19 + 24 - 6 = 43 - 6 = \boxed{37}$$

Evaluate $\begin{vmatrix} 2 & -3 & 4 \\ 1 & 5 & -2 \\ 3 & 2 & 2 \end{vmatrix}$

Always

$$= 2\begin{vmatrix} 5 & -2 \\ 2 & 2 \end{vmatrix} - (-3)\begin{vmatrix} 1 & -2 \\ 3 & 2 \end{vmatrix} + 4\begin{vmatrix} 1 & 5 \\ 3 & 2 \end{vmatrix}$$

$$= 2(5\cdot 2 - 2\cdot(-2)) + 3(1\cdot 2 - 3(-2)) + 4(1\cdot 2 - 3\cdot 5)$$

$$= 2(10+4) + 3(2+6) + 4(2-15)$$

$$= 2(14) + 3(8) + 4(-13)$$

$$= 28 + 24 - 52 = 52 - 52 = \boxed{0}$$

Evaluate

Always

$$\begin{vmatrix} 2 & -5 & -4 \\ 0 & -3 & 7 \\ 0 & 0 & 5 \end{vmatrix} = 2\begin{vmatrix} -3 & 7 \\ 0 & 5 \end{vmatrix} - (-5)\begin{vmatrix} 0 & 7 \\ 0 & 5 \end{vmatrix} + (-4)\begin{vmatrix} 0 & -3 \\ 0 & 0 \end{vmatrix}$$

$$= 2(-3\cdot 5 - 0\cdot 7) + 5(0\cdot 5 - 0\cdot 7) - 4(0\cdot 0 - 0\cdot(-3))$$

$$= 2(-15) + 5(0) - 4(0)$$

$$= -30 + 0 - 0 = \boxed{-30}$$

Cramer's Rule for solving system of linear equations with 3 unknowns and 3 equations.

$$\begin{cases} x + 2y - z = -4 \\ x + 4y - 2z = -6 \\ 2x + 3y + z = 3 \end{cases} \qquad \left.\begin{array}{l} x = \dfrac{D_x}{D} \\ y = \dfrac{D_y}{D} \\ z = \dfrac{D_z}{D} \end{array}\right\} D \neq 0$$

Always

$$D = \begin{vmatrix} 1 & 2 & -1 \\ 1 & 4 & -2 \\ 2 & 3 & 1 \end{vmatrix} = 1\begin{vmatrix} 4 & -2 \\ 3 & 1 \end{vmatrix} - 2\begin{vmatrix} 1 & -2 \\ 2 & 1 \end{vmatrix} + (-1)\begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix}$$

$$= 1(4-(-6)) - 2(1-(-4)) - 1(3-8)$$

$$= 1\cdot 10 - 2\cdot 5 - 1\cdot(-5) = \boxed{5}$$

Always

$$D_z = \begin{vmatrix} 1 & 2 & -4 \\ 1 & 4 & -6 \\ 2 & 3 & 3 \end{vmatrix} = 1\begin{vmatrix} 4 & -6 \\ 3 & 3 \end{vmatrix} - 2\begin{vmatrix} 1 & -6 \\ 2 & 3 \end{vmatrix} + (-4)\begin{vmatrix} 1 & 4 \\ 2 & 3 \end{vmatrix}$$

$$= 1(12-(-18)) - 2(3-(-12)) - 4(3-8)$$

$$= 30 - 30 + 20 = \boxed{20}$$

$$z = \frac{D_z}{D} = \frac{20}{5} = 4 \qquad \boxed{z = 4}$$

Use Cramer's rule to solve for y only:

$$\begin{cases} 3x - 2y + z = 10 \\ 2x + 3y - z = -9 \\ x + 4y + 3z = 2 \end{cases} \qquad y = \frac{D_y}{D}$$

Always

$$D = \begin{vmatrix} 3 & -2 & 1 \\ 2 & 3 & -1 \\ 1 & 4 & 3 \end{vmatrix} = 3\begin{vmatrix} 3 & -1 \\ 4 & 3 \end{vmatrix} - (-2)\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 2 & 3 \\ 1 & 4 \end{vmatrix}$$

$$= 3(9+4) + 2(6+1) + 1(8-3)$$

$$= 3(13) + 2(7) + 1(5)$$

$$= 39 + 14 + 5 = \boxed{58}$$

$$D_y = \begin{vmatrix} 3 & 10 & 1 \\ 2 & -9 & -1 \\ 1 & 2 & 3 \end{vmatrix} = 3\begin{vmatrix} -9 & -1 \\ 2 & 3 \end{vmatrix} - 10\begin{vmatrix} 2 & -1 \\ 1 & 3 \end{vmatrix} + 1\begin{vmatrix} 2 & -9 \\ 1 & 2 \end{vmatrix}$$

$$= 3(-27+2) - 10(6+1) + 1(4+9)$$

$$= 3(-25) - 10(7) + 1(13)$$

$$= -75 - 70 + 13 = \boxed{-132}$$

$$y = \frac{D_y}{D} = \frac{-132}{58} = \boxed{-2.275}$$

The sum of 3 numbers is 6.
x, y, z

Twice the first one plus the second one is 4.
$2x \quad + \quad y$

The difference of first and third is -2.
Use Cramer's rule to find the first one only.

$$\begin{cases} x + y + z = 6 \\ 2x + y = 4 \\ x - z = -2 \end{cases} \qquad x = \frac{D_x}{D}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = 1\begin{vmatrix} 1 & 0 \\ 0 & -1 \end{vmatrix} - 1\begin{vmatrix} 2 & 0 \\ 1 & -1 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ 1 & 0 \end{vmatrix}$$

$$= 1(-1-0) - 1(-2-0) + 1(0-1)$$

$$= -1 + 2 - 1 = \boxed{0}$$

Since $D = 0 \Rightarrow$ Cramer's rule cannot be applied.

Graph of $y = ax^2 + bx + c$ contains $(1,3)$, $(-1,9)$, and $(2,6)$.
Use Cramer's rule to find a.

$(1,3) \rightarrow \begin{matrix} x=1 \\ y=3 \end{matrix} \rightarrow a(1)^2 + b(1) + c = 3 \rightarrow a + b + c = 3$

$(-1,9) \rightarrow \begin{matrix} x=-1 \\ y=9 \end{matrix} \rightarrow a(-1)^2 + b(-1) + c = 9 \rightarrow a - b + c = 9$

$(2,6) \rightarrow \begin{matrix} x=2 \\ y=6 \end{matrix} \rightarrow a(2)^2 + b(2) + c = 6 \rightarrow 4a + 2b + c = 6$

$$\begin{cases} a + b + c = 3 \\ a - b + c = 9 \\ 4a + 2b + c = 6 \end{cases} \Rightarrow a = \frac{D_a}{D}$$

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 4 & 2 & 1 \end{vmatrix} = 1\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 1 & 1 \\ 4 & 1 \end{vmatrix} + 1\begin{vmatrix} 1 & -1 \\ 4 & 2 \end{vmatrix}$$

$$= 1(-1-2) - 1(1-4) + 1(2+4)$$

$$= -3 + 3 + 6 = \boxed{6}$$

$$D_a = \begin{vmatrix} 3 & 1 & 1 \\ 9 & -1 & 1 \\ 6 & 2 & 1 \end{vmatrix} = 3\begin{vmatrix} -1 & 1 \\ 2 & 1 \end{vmatrix} - 1\begin{vmatrix} 9 & 1 \\ 6 & 1 \end{vmatrix} + 1\begin{vmatrix} 9 & -1 \\ 6 & 2 \end{vmatrix}$$

$$= 3(-3) - 1(3) + 1(24)$$

$$= -9 - 3 + 24 = \boxed{12}$$

$$a = \frac{D_a}{D} = \frac{12}{6} = \boxed{2}$$

I have 10 coinst. Quarters and Dimes only.

Total value \$1.60. How many Quarters do I have?

$$\begin{cases} Q + D = 10 \\ 25Q + 10D = 1\ 60 \end{cases} \div 5 \Rightarrow \begin{cases} Q + D = 10 \\ 5Q + 2D = 32 \end{cases}$$

$$-5\left[\begin{array}{cc|c} 1 & 1 & 10 \\ 5 & 2 & 32 \end{array}\right]$$

$(-5)R1 + R2 \to R2$

$$\left[\begin{array}{cc|c} 1 & 1 & 10 \\ 0 & -3 & -18 \end{array}\right]$$

$R2 \div (-3) \to R2$

$$\left[\begin{array}{cc|c} 1 & 1 & 10 \\ 0 & 1 & 6 \end{array}\right]$$

$(-)R2 + R1 \to R1$

$$\left[\begin{array}{cc|c} 1 & 0 & 4 \\ 0 & 1 & 6 \end{array}\right] \quad Q=4,\ D=6$$

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Consider

$$\begin{cases} 5x + 3y = 7 \\ -x + 2y = 9 \end{cases}$$

Use Cramer's rule to find y only. $y = \frac{D_y}{D}$

$$D = \begin{vmatrix} 5 & 3 \\ -1 & 2 \end{vmatrix} = 5(2) - (-1)(3) = 10 + 3 = \boxed{13}$$

$$D_y = \begin{vmatrix} 5 & 7 \\ -1 & 9 \end{vmatrix} = 5(9) - (-1)(7) = 45 + 7 = \boxed{52}$$

$$y = \frac{D_y}{D} = \frac{52}{13} = \boxed{4}$$

$$\boxed{y = 4}$$